Arbitrage

1. Exchange rate arbitrage
Exchange rate arbitrage is the practice of taking advantage of inconsistent exchange rates in different markets by selling in one market and simultaneously buying in another. Arbitrageurs do not take risks or, at least, it is not their intention to do so. In other words, they endeavour to maintain closed positions at all times. Rates of profit on arbitrage operations are necessarily low in competitive, well-informed markets, but since transactions are usually very large, absolute profits may also be large from successful arbitrage. Arbitrage performs the function for a market system of bringing prices in one market into line with those in other markets.

There are two types of arbitrage of relevance to forex markets: exchange rate arbitrage and interest rate arbitrage. In exchange rate arbitrage, advantage is taken of differentials in the price of a currency in different markets. Exchange rate arbitrage transactions may be classified in terms of the number of markets involved. Thus, we may have two-point and three-point arbitrage.

1.1 Two-point arbitrage
Two-point arbitrage concerns two currencies in two geographically separated markets. For example, let the spot exchange rate be £1 = $1.55 in London and £1 = $1.60 in New York. Here we are quoting both exchange rates against sterling. That is, we are quoting GBP/USD. This is the indirect quotation of sterling and the direct quotation of the dollar. Remember that the expression Currency A/Currency B gives you the amount of Currency B that exchanges for one unit of currency A. In practice, most exchange rates are quoted against the US dollar. If we were to do this, we would quote:

In London: USD/GBP £0.645
In New York: USD/GBP £0.625.

Thus, in relative terms, sterling is undervalued in London and overvalued in New York. Provided that capital was free to flow between the two centres, arbitrageurs would attempt to exploit, and hence profit from, the differential by selling dollars for pounds in London and reselling the pounds in New York. Assume the arbitrageur sold $1 million in London. For this, he would have received £645,161.29. Selling this in New York would have returned him £1,032,258.06 - a profit of 5 cents per £1. The sale of dollars in London would have strengthened sterling and pushed the value of the pound above $1.55. At the same time, the sale of sterling in New York would have caused sterling to weaken there, pushing its value below $1.60. The action of arbitrageurs would bring the rates of exchange in the two centres together.

In practice, the rates wouldn't come exactly into line because of the existence of transactions costs, but the rates should move to being 'transactions costs close'. There is another simplification in the above example since no regard is paid to the existence of bid and offer rates of exchange. In the real world, the rates may have been something like:

<table>
<thead>
<tr>
<th>London: GBP/USD</th>
<th>Bid</th>
<th>Offer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.5495</td>
<td>1.5505</td>
</tr>
</tbody>
</table>

Selling dollars in London, the arbitrageur would have been quoted the offer rate of 1.5505 and, thus, would have received £644,953.24. Buying dollars in New York, the arbitrageur would have been quoted the bid rate of 1.5995 and would have received £1,031,602.71. That is, the profits would have been lower because of the bid-offer spread.
1.2 Three-point (triangular) arbitrage
Exchange rates may be externally consistent but internally inconsistent. That is, exchange rates among different currencies may be mutually inconsistent. Arbitrageurs will then attempt to profit from these inconsistencies and in the process will eliminate discrepancies and establish mutually consistent cross-exchange rates. A cross-exchange rate is simply the price of a second currency expressed in terms of a third or an exchange rate calculated from two other rates. For example, the rate of the Euro against the Swedish krona derived as the cross rate from US$ - Krona and US$ against the Euro.

Consider the following market rates:
USD/CHF (Swiss francs against the US dollar) 1.6639-46 ($1 = SwFr 1.6639-1.6646)
EUR/USD (US dollar against the Euro) 0.9682-86 (€1 = $0.9682-0.9686)

Let us use these two exchange rates to calculate the cross-rate of exchange of Swiss francs against the Euro. To do this, we must multiply the same side of each exchange rate. Thus we get:

\[
1.6639 \times 0.9682 \text{ gives us Bid rate } €1 = \text{ SwFr} 1.6110
\]
\[
1.6646 \times 0.9686 \text{ gives us Offer rate } €1 = \text{ SwFr} 1.6123
\]

That is, we have:  EUR/CHF  1.6110-23

If you wish to see how this works, you can work through the various stages. The bid rate is the rate at which the bank bids for Euros. That is, the rate at which the bank buys Euros. Therefore, it applies to the case where a client is selling Euros to the bank in exchange for Swiss dollars. Therefore, start with €1,000,000 and sell them for US dollars. The bank will buy dollars for Euros at a rate of €1 = 0.9682 and the client obtains $968,200. Then, the client sells dollars for Swiss francs. The bank is now buying dollars and will pay only SwFr1.6639 for each dollar and the client obtains SwFr1,610,988, which gives a rate of exchange of €1 = SwFr1.6110 when rounded to four decimal places.

The offer rate is the rate at which the bank sells (offers) Euros. Therefore, this applies to the case where the client is selling Swiss francs and buying Euros. Now we start with SwFr 1 million and sell them for dollars. The bank will demand SwFr1.6646 for each dollar and so we have $600,744.92. We then sell them for Euros but the bank now will pay us only €1/0.9686 for each dollar (that is, €1.0324). Therefore, we need to divide $600,744.92 by 0.9686 and this gives us €620,219.82 and we have an exchange rate of SwFr1 = €0.62022. However, we wish to know how many Swiss francs we can get for one Euro and so we need the reciprocal of this, which is 1.6123.

This particular way of calculating the cross-rate (multiplying the same side of each exchange rate) is needed because we started with one rate in which the dollar was quoted indirectly (USD/CHF) and one where the dollar was quoted directly (EUR/USD). Had we started off with two indirect quotations, the process would have been different. We would have had:

USD/CHF  1.6639-46
USD/EUR  1.0324-28

Then we would have had to cross-divide the two exchange rates:

\[
1.0324 \div 1.6646 = 0.6207 \text{ and the reciprocal is 1.6110}
\]
and \[
1.0324 \div 1.6646 = 0.6202, \text{ whose reciprocal is 1.6123.}
\]
If both dollar rates had been quoted directly, we would have had:

CHF/USD 0.6007-10
EUR/USD 0.9682-86

We would then have followed the steps:

0.6007 ÷ 0.9686 = 0.6202
0.6010 ÷ 0.9682 = 0.6207

These complications arise because the lower number is always placed first when quoting bid/offer spreads. In fact, virtually all cross-rates are calculated through the US dollar and the US dollar is normally quoted indirectly.

With all this consideration of cross-rates of exchange, we have rather lost sight of three-point arbitrage. In our first calculation, we finished up with three rates of exchange:

USD/CHF 1.6639-46
EUR/USD 0.9682-86
EUR/CHF 1.6110-23

Suppose now that an inconsistency developed and that the EUR/CHF rate changed to 1.6410-23. That is, in relation to the cross-rate, the Euro was overpriced in terms of Swiss francs. There would be a profit opportunity for arbitrageurs. The question is how one would go about exploiting it (do things the wrong way round and you will make a loss).

It is easy enough to work out the correct procedure. All you need to do is to remember that, to make money, you buy cheap and sell dear. Therefore, you wish always to be selling overpriced products, not buying them. Therefore, to profit in the case above you need to be selling Euros and buying Swiss francs. Then, it depends on which currency you start.

- If you start in dollars, you must sell the dollars for Euros, use the Euros to buy Swiss francs and sell the Swiss francs for dollars.

- If you start in Swiss francs, you need to sell them for dollars, use the dollars to buy Euros and then use the Euros to buy Swiss francs.

- If you start in Euros, you must use the Euros to buy Swiss francs, use the Swiss francs to buy dollars and then use the dollars to buy Euros.

We can check by working out the figures for one of these. Assume we start with $1 million. We sell dollars and buy Euros at €1 = 0.9686 and this gives us €1,032,417.92. We sell these for Swiss francs at SwFr1.6410, giving us SwFr1,694,197.81. We sell these for dollars at 1.6646 and finish up with $1,017,780.74. Thus, we have made a profit. You will get the same answer if you start with Euros or Swiss francs and follow the order of the steps above. The crucial point is that you must always, as one of the steps be selling Euros for Swiss francs. Of course, if the market rate were below the cross-rate of exchange, you would need to sell Swiss francs for Euros in order to make a profit.

As with any other arbitrage operation, the sale and purchase of currencies will change the exchange rates until the possibility of profit disappears.

Exercise 1: Imagine you are a British arbitrageur, holding sterling, in the following example:
Actual exchange rates

<table>
<thead>
<tr>
<th>Currency Pair</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>GBB/USD</td>
<td>£1 = $1.5715-721</td>
</tr>
<tr>
<td>USD/JPY</td>
<td>$1 = ¥106,090-120</td>
</tr>
<tr>
<td>GBP/JPY</td>
<td>£1 = ¥176,720-831</td>
</tr>
</tbody>
</table>

Start with £1,000,000.

(a) List the steps you need to take to make a profit.

(b) Calculate the rate of profit you will make.

**Answer to Exercise 1**

Implied cross rates are £1 = ¥166,720-831. Thus, in the actual market, sterling is overpriced in relation to yen and we must sell sterling for yen.

Thus:
Step A: Use £ to buy yen; Step B: Use yen to buy $; Step C: Use $ to buy £

Step A: Sell £ for yen; market-maker sells the foreign currency (¥) at the bid rate of ¥176,720. This gives ¥176,720,000.
Step B: Sell ¥ for $; market-maker sells dollars at the offer rate of ¥106.120. This gives $1,665,284.58
Step C: Sell $ for £. The market-maker buys dollars at the higher rate of $1.5721, which gives £1,059,273.95 or a profit of 5.9%.

This is, of course, a ridiculously high rate of profit, since it could be made in a matter of moments. In practice, rates only have to get slightly out of line before the arbitrageurs step in.

**2. Interest rate arbitrage**

This is the simultaneous exploitation of interest rate differentials in one or more markets for profit. Arbitrageurs may either transfer their own capital from one market to another or simultaneously borrow in one market and lend in another. Interest rate arbitrage is based upon the principle of interest rate parity, which, as we have seen in our discussion of forward exchange rates, is expressed by: the difference in interest rates = difference between spot and forward rates of exchange:

\[
\frac{f_0 - s_0}{s_0} = \frac{i_£ - i_€}{1 + i_£}
\]

where \(f_0\) is the forward rate of exchange of the foreign currency (euros) against the home currency (sterling); \(s_0\) is the spot rate of exchange between the two currencies; \(i_£\) is the interest rate on the foreign currency; and \(i_€\) is the interest rate on the home currency. Consider the following interest arbitrage example.

An investor has £1m to invest for 12 months. GBP/EUR exchange rates are:
Spot £1 = €1.6231 (taking the mid-point between the bid and offer rates); 12 months' forward £1 = €1.5871. Money market interest rates for 12 months are: on sterling 6.81%; on the Euro 4.28%.

The sterling investment would yield £1,068,100; the € investment plus the forward purchase of £ gives £1,066,453. Allow for transactions costs (which we have not done since we have ignored the bid-offer spreads) and one can see that there are no arbitrage possibilities here.

Another way of checking is to put the figures into the above equation. Then,

\[
LHS = \frac{(f_0 - s_0)}{s_0} = -0.0222
\]

\[
RHS = \frac{(i_€ - i_£)}{(1 + i_£)} = \frac{(0.0428 - 0.0681)}{1.0681} = -0.0236
\]
Again, the results are transactions-cost close. The LHS (the annual forward premium) is known as the exchange agio. The negative signs tell us that sterling is at a forward discount. The RHS is called the interest agio. If there are no opportunities for profitable covered interest arbitrage, the interest agio will equal the exchange rate agio, allowing for transactions costs.

However, if these figures differed then covered interest arbitrage could be possible. Assume the same spot exchange rate but the following interest rates. Sterling 6.81% Euro 3.58%. Now, the interest agio (the interest rate differential) = \((0.0358 - 0.0681)/1.0681 = -0.0302\). Suppose we have a large sum in £ and we wish to invest this for a short period of time in low-risk securities, say for three months. We have two choices:

(a) buy securities in the UK at UK interest rates;
(b) change sterling into Euros; buy Eurozone securities and receive Eurozone interest rates and, when the securities mature, convert the Euros back into £.

The second choice, however, involves a risk - the risk that during the three-month period, the value of the Euro will fall against sterling. If we wish to avoid risk (that is, we wish to maintain a closed position), we can, at the same time as we buy the Euro securities, sell the amount of Euros we know we shall have at the end of the three months forward at an agreed rate of exchange. Then, when our securities mature, we meet our forward contract and convert the Euros back into sterling. As a result, we know in advance which of (a) or (b) will be the more profitable. The 3-month forward rate is £1 = €1.6136.

In the above example, it is clear that (a) will be more profitable. Why? Let us work it out first. Assume we start with £1 million. Investing this at 6.81% per annum for three months will return us £1,017,025. If we change £ into Euros at the spot rate, we receive €1,623,100. Investing this at 3.58% for three months gives us €1,637,627 and converting this back into £ at the previously agreed forward rate of £1 = €1.6136 gives £1,014,890. OK, (a) is more profitable? Why? Given the size of the interest rate differential the forward discount on sterling should have been greater than it was. The three-month exchange agio was: \((1.6136 - 1.6231)/1.6231 = \text{That is, } -0.00585\), much smaller than the interest agio of -.0302. Clearly, the value of sterling spot and forward has to change or interest rates have to change.

In fact, both will happen. UK residents buy sterling bonds, helping to push up UK bond prices and push down UK interest rates. Eurozone residents sell European bonds, pushing their prices down and putting upward pressure on Euro interest rates. With the proceeds from their sale of bonds, Eurozone residents buy sterling spot in order to purchase UK bonds and sell sterling forward to protect themselves from exchange rate risk. The spot exchange rate of sterling rises above €1.6231 and the forward rate falls below €1.6136. The interest rate gap narrows and the spot/forward spread widens until the interest agio and the exchange agio are transactions-cost close.

Of course, we could have seen the problem easily from the pages of the Financial Times, without doing any calculation. We were told in the newspaper that, on the day in question (18 March 2000), the Euro was trading three-months forward against sterling at a premium of 2.4% per annum. That was fairly close to the actual interest rate difference of 2.53% (6.81 - 4.28). Then, we assumed that Euro interest rates fell to 3.58%, increasing the interest rate difference to 3.23%. It is thus very clear that, under these circumstances, there would have been a profitable arbitrage operation and that the result of that operation would have been to increase the forward premium on the Euro. That is, the arbitrage operation would have had to involve buying sterling spot and selling sterling forwards.
2.1 A very informal check on covered interest rate parity

Consider the following table:

<table>
<thead>
<tr>
<th>Currency</th>
<th>1-month interest rate on £</th>
<th>1-month interest rate on foreign currency</th>
<th>interest rate differential</th>
<th>spot rate of exchange</th>
<th>1-month forward rate of exchange</th>
<th>1-month forward premium or discount (+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro</td>
<td>6.03</td>
<td>3.61</td>
<td>2.42</td>
<td>1.6231</td>
<td>1.6195</td>
<td>2.7</td>
</tr>
<tr>
<td>Swiss franc</td>
<td>6.03</td>
<td>2.24</td>
<td>3.79</td>
<td>2.6159</td>
<td>2.6065</td>
<td>4.3</td>
</tr>
<tr>
<td>US $</td>
<td>6.03</td>
<td>6.06</td>
<td>-0.003</td>
<td>1.5718</td>
<td>1.5721</td>
<td>-0.2</td>
</tr>
<tr>
<td>Yen</td>
<td>6.03</td>
<td>0.20</td>
<td>5.83</td>
<td>166.766</td>
<td>165.901</td>
<td>6.3</td>
</tr>
</tbody>
</table>

The figure in the middle column, the interest rate differential, would represent perfect interest rate parity, on the assumption that:

(a) there were no transactions costs; and
(b) the securities of different countries were perfect substitutes for each other.

We can see that the actual premium/discount is in all cases larger than the interest rate differential, but this is what we would expect simply from the existence of transactions costs. Remember that we are here using exchange rates that are the mid-points between bid and offer rates. Allowing for this, we can see that the forward premiums/discounts are sufficiently close to the interest rate differential for us to conclude that this differential is the major influence on the relationship between spot and forward exchange rates and that covered interest rate parity holds. In other words, arbitrageurs have acted to take advantage of any possible arbitrage opportunities and, in doing so, have removed any market inconsistencies.

2.2 Algebraic expression of interest rate parity

A US exporter due to receive £A in one year (at time $t$ where $t = 12$ months) might avoid foreign risk in one of two ways:

Using the forward market, his US $ proceeds at time $t$ would be $f_oA$. Alternatively, he could borrow $[£A/(1 + i£)]$ and convert it into $US$ giving him $[£A/(1 + i£)].s_o$. Investing this now would yield at time $t$ the sum of:

$$[£A/(1 + i£)].s_o(1 + i_s).$$

Assuming equilibrium between money markets and foreign exchange markets, this must equal $f_oA$. That is:

$$[£A/(1 + i£)].s_o(1 + i_s) = f_oA$$

Dividing by A and re-arranging gives:

$$f_o = s_o[(1 + i_s)/(1 + i£)]$$
Dividing by $s_0$ and deducting 1 gives:

\[(f_0/s_0) - 1 = [(1 + i$_S$)/(1 + i$_£$)] - 1\]

That is,

\[(f_0 - s_0)/s_0 = [(1 + i$_S$)/(1 + i$_£$)] - 1\]

and,

\[(f_0 - s_0)/s_0 = (i$_S$ - i$_£$)/(1 + i$_£$)\]

We can generalize this by letting the US $ = k$ and £ = j and writing:

\[(F - E)/E = (i_k - i_j)/(1 + i_j)\]

Interest rates are always expressed in decimals. F and E in this expression are the forward and spot exchange rates expressed as units of k's currency/units of j's currency i.e. the price of the foreign currency.